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Modeling A Mathematical Solution for the Optimization of Signal-to-Background Ratio

Ding Lee
Donna F. Yarger
Systems Analysis Department

8 December 1977

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PREFACE

This research was performed under NUSC Overhead No. 720B16.

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One of the complex and unsolved problems in the sonar field is the modeling of a mathematical and computerized solution for the maximization of signal-to-background ratio (SBR). In dealing with the maximization of SBR, it has been found sufficient to attack the problem of maximizing signal-to-reverberation ratio (SRR). In this report, a numerical mathematical model is introduced for the complete solution of the SRR-maximization problem. The techniques employed to develop the entire optimization process are fully (over)

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20. ABSTRACT (Continued)

justified by mathematical theory. A mathematical expression for SRR is obtained, the complete SRR-optimization problem is addressed, and the development of a systematic computerized numerical procedure to optimize SRR is outlined. Also, the problems encountered in the mathematical and computer modeling are discussed. The computerized model has been applied to several sonar problems. Numerical results have demonstrated the validity of the mathematical model and the effectiveness of the computer model.

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MODELING A MATHEMATICAL SOLUTION FOR THE OPTIMIZATION OF SIGNAL-TO-REVERBERATION RATIO

INTRODUCTION

A sonar system uses acoustic energy for the purposes of detection, distance estimation, and communications. A passive sonar receives acoustic energy generated by a target, whereas an active sonar generates acoustic energy that is reflected by a target and detected by the sonar receiver.

There is a desired and an undesired acoustic energy associated with the natural acoustical environment. The desired portion is the signal and the undesired portion is the background. The background is either ambient noise or reverberation noise. The main objective of research is to increase sonar performance, which means increasing the overall response of the sonar system to the signal and decreasing the overall response of the sonar system to the background, i.e., increasing the signal-to-background ratio (SBR).

One of the complex and unsolved sonar problems is the modeling of a computerized mathematical solution for the optimization of the SBR. Since optimization of signal-to-noise ratio (SNR) is a special case of the optimization of signal-to-reverberation ratio (SRR), it is sufficient to consider the general SRR problem in dealing with the optimization of SBR. Much of the existing theory and techniques that have been developed have been directed at maximizing SNR, leaving unfinished a comprehensive treatment of the general case. In general, reverberation produced by the scatterers is not identical; therefore, the distribution is not uniform. This presents a formulation difficulty so that an exact mathematical expression for the SRR is not easy to obtain. However, under some realistic environmental conditions, reasonable assumptions can be made in order to make the calculation of SRR by a mathematical expression possible. From this mathematical expression, a complete definition of SRR will be established in order to classify SBR in different categories. The expression, a two-dimensional integral, is difficult to evaluate in general. In many applications the solutions are special functions, such as Bessel functions. In others, a closed-form mathematical solution is extremely difficult to obtain. Reverberation power received from the array is positive; this physical property allows the application of the quadratic-form theory to optimize SRR. Note that other techniques used to maximize SNR, such as Lagrange Multipliers,[1] matching filter technique,[2] statistical techniques,[3] and others,[4-14] are all applicable to maximize SRR. However, this model uses the elegant

quadratic-form theory because it yields an exact formulation for the systematic determination of signal and reverberation matrices that is very suitable for any computer. Under general conditions, when the mathematical expression cannot be obtained, background measurements can be used together with the quadratic-form theory to optimize SRR.

A complete computerized procedure that implements the theory will be outlined in this report. A detailed description of the computer model will be reported separately. A natural question arises: Can this combined procedure be improved? A discussion of the problems encountered and some practical solutions may serve as an answer to the above question. A sizeable cylindrical array is presented as an example to illustrate the problems encountered and to show how this problem was solved practically. Experimental computer programs, designed to implement this procedure, are written in ANSI FORTRAN language, but will not be included in this report. However, the finalized package of programs will be included in the computer model report.

A MATHEMATICAL MODEL TO MAXIMIZE SRR

In this report, d_j, R_j, and R are functions of θ,ϕ; U, V are functions of θ,ϕ, and α_j. For economy in wording, we have dropped the symbols θ,ϕ, and α_j for the above functions.

BEAM OUTPUT FUNCTION AND PHYSICAL CONSIDERATIONS

Figure 1 shows a configuration in three-dimensional space with spherical angles θ,ϕ, which have the following definitions:

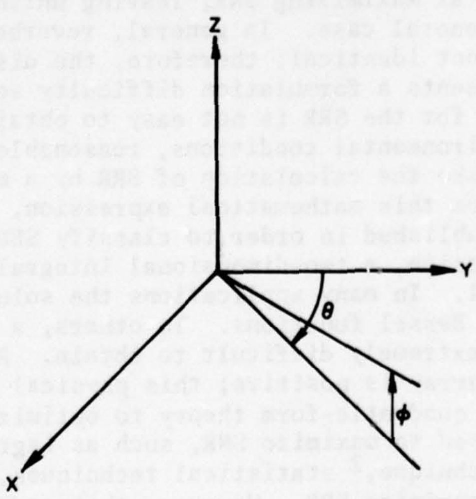

Figure 1. Three-Dimensional Space With Spherical Angles θ,ϕ

let $\theta (0 \le \theta \le 2\pi)$ define the azimuthal deviation in the horizontal plane;

let $\phi (-\frac{1}{2}\pi \le \phi \le \frac{1}{2}\pi)$ define the vertical deviation from a horizontal reference;

let $R_j (\theta,\phi)$ (or R_j) be the amplitude response of the j-th element of a sonar array consisting of n elements;

let $d_j (\theta,\phi)$ (or d_j) be the distance from the j-th element to the reference plane perpendicular to the direction (θ,ϕ) of the incoming acoustic wave; and

let λ be the wavelength.

Distances d_j are obtained by means of direction cosines that define the direction of propagation perpendicular to the incoming wavefront. The beam output function $V(\theta,\phi,\alpha_j)$ (or V) is defined by

$$V = \sum_{j=1}^{n} \alpha_j R_j e^{i\left(\frac{2\pi}{\lambda}\right)d_j}, \tag{1}$$

where α_j are scalars to be determined.

Associated with each sonar system there is a beam pattern that is calculated by means of the beam output function. An important sonar parameter for sound reception used to measure the amount of response through the beam pattern and to discriminate against noise in favor of signal is conventionally termed directivity index (DI). DI refers to a plane-wave signal in an isotropic noise field.[15] These physical conditions permit DI to be expressed mathematically by the two-dimensional integral given by equation (6). When signal and noise fields have other directionalities, the term "array gain" (AG) is used. The mathematical expression of AG is given by equation (5). The definition of AG is, then, generalized to obtain a mathematical expression for reverberation gain (RG) given by equation (3).

MATHEMATICAL EXPRESSIONS

In the region $0 \le \theta \le 2\pi$, $-\frac{1}{2}\pi \le a \le \phi \le b \le \frac{1}{2}\pi$, a generalized directive gain (D) can be defined as follows:

$$D = \frac{|V_0|^2}{\frac{1}{2\pi(\sin b - \sin a)} \int_0^{2\pi} \int_a^b R \frac{|U|^2}{|U_0|^2} |V|^2 \cos \phi \, d\phi \, d\theta}, \tag{2}$$

where

$R = R(\theta,\phi)$ for an active sonar system defines a reverberation function that is used to characterize the relative reverberation strength of backscattered energy from boundary or volume scatterers over the angular interval of interest, and for a passive sonar system defines a noise function;

$U = U(\theta,\phi)$ defines a transmitting-beam output function;

$U_0 = U(\theta_0,\phi_0)$ defines a transmitting steering-direction vector;

$V = V(\theta,\phi,\alpha_j)$ defines a receiving-beam output function; and

$V_0 = V(\theta_0,\phi_0,\alpha_j)$ defines a receiving steering-direction vector.

In equation (2), let D be D_1 when R = 1: let D be D_2 when U = 1, $a = -\frac{1}{2}\pi$, $b = \frac{1}{2}\pi$; let D be D_3 when R = 1, U = 1, $a = -\frac{1}{2}\pi$, $b = \frac{1}{2}\pi$. The terms reverberation gain RG, reverberation index RI, array gain AG, and directivity index DI then have the following mathematical definitions:

$$\text{RG (reverberation gain)} = 10 \log_{10} D, \tag{3}$$

$$\text{RI (reverberation index)} = 10 \log_{10} D_1, \tag{4}$$

$$\text{AG (array gain)} = 10 \log_{10} D_2, \text{ and} \tag{5}$$

$$\text{DI (directivity index)} = 10 \log_{10} D_3. \tag{6}$$

It is seen that RG in equation (3) is the most general expression, and that RI, AG, and DI are special cases of RG. This shows that in dealing with the SBR-maximization problem it is sufficient to deal with the maximization of equation (3), which is equivalent to the maximization of D in equation (2). Equations (3) and (4) are valid for active sonars with reverberation background, and equations (5) and (6) are valid for passive sonars with noise background.

The two-dimensional integral, occurring in equation (2), is usually not simple to evaluate. In some applications, the exact solutions exist in closed forms; in this case, the expression is usually a combination of Bessel functions.[16] When the closed-form solutions cannot be obtained, numerical integration techniques are used.

THE SRR-MAXIMIZATION PROBLEM AND A FEASIBLE SOLUTION

Under all conditions, it is desirable to increase the signal-to-background ratio (SBR). For the situations where ambient noise is the dominant interference, the maximization of SNR is required. A closely related and more general problem arises when interference is dominated by reverberation; in this situation the maximization of SRR is required. In dealing with the maximization of SBR, it is sufficient to deal with the maximization of SRR.

The SRR-maximization problem can be described by one question: Can a set of scalar coefficients αj be determined such that the generalized directive gain D of equation (2) is at its maximum? The answer is yes, if D possesses the necessary properties. The reverberation power received by the array is always positive; this implies that the denominator of D is positive. This property allows D to be expressed as an SRR and suggests that D can be expressed simultaneously as a ratio of two quadratic forms with a positive-definite denominator; therefore, the elegant quadratic-form theory can be applied to maximize SRR. The quadratic-form theory not only gives the exact mathematical formulation and the optimum solution to the SRR-maximization problem, but also gives a systematic optimization procedure that is very suitable for implementation on large-scale computers. The approach is to express

$$D = \frac{\text{signal}}{\text{reverberation}} = \frac{\boldsymbol{\alpha}^* \mathbf{A} \boldsymbol{\alpha}}{\boldsymbol{\alpha}^* \mathbf{B} \boldsymbol{\alpha}} , \tag{7}$$

where * indicates conjugate transpose.

Setting $\boldsymbol{\alpha}^* \mathbf{A} \boldsymbol{\alpha} = |V_0|^2$, we can express the n^2 elements of $\mathbf{A} = (a_{jk})$ by

$$a_{jk} = e^{i\frac{2\pi}{\lambda}[d_k(\theta_0,\phi_0)-d_j(\theta_0,\phi_0)]} , \tag{8}$$

for $j,k = 1,2,\ldots,n$.

Since a steering direction is given, without loss of generality, we drop the amplitude for V for formulation simplicity and define a steering direction column vector $\mathbf{E}$ to be

$$\mathbf{E} = e^{-i\frac{2\pi}{\lambda}[d_j(\theta_0,\phi_0)]} , \tag{9}$$

for j = 1,2,...,n. A simple verification shows that

$$\mathbf{E}\mathbf{E}^* = \mathbf{A} \quad . \tag{10}$$

Equation (10) gives an alternate determination of a_{jk}. Similarly, setting $\alpha^*\mathbf{B}\alpha$ equal to the denominator of $\mathbf{D}$, the n^2 elements of $\mathbf{B} = (b_{jk})$ are expressed by

$$b_{jk} = \frac{1}{2\pi(\sin b - \sin a)} \int_0^{2\pi}\!\!\int_a^b R \frac{|U|^2}{|U_0|^2} e^{i\frac{2\pi}{\lambda}(d_k - d_j)} \cos\phi \, d\phi d\theta \, , \tag{11}$$

where $\alpha^*\mathbf{B}\alpha$ is positive because of the physical property; this implies $\mathbf{B}$ is positive-definite. Then D can be maximized according to the following matrix theory.

If $\mathbf{A}(\alpha,\alpha)$ and $\mathbf{B}(\alpha,\alpha)$ are 2 quadratic forms in the variable α_j and $\mathbf{B}$ is positive-definite, then the characteristic equation of $\mathbf{A}(\alpha,\alpha) - \lambda\,\mathbf{B}(\alpha,\alpha)$ is

$$p(\lambda) = \det(\mathbf{A} - \lambda\mathbf{B}) = 0 \quad . \tag{12}$$

In view of the matrix theory for eigenvalue problems,[17] $p(\lambda)$ has real roots λ_j such that $\lambda_j \le \lambda_{j+1}$ for j = 1,2,...,n-1 and λ_1, λ_n represent the bounds of equation (7); i.e.,

$$\lambda_1 \le \frac{\alpha^*\mathbf{A}\alpha}{\alpha^*\mathbf{B}\alpha} \le \lambda_n \quad . \tag{13}$$

The right equality of equation (13) is attained if

$$\mathbf{A}\alpha = \lambda_n \mathbf{B}\alpha \quad . \tag{14}$$

Then, it is seen that

$$\lambda_n = \frac{\alpha^*\mathbf{A}\alpha}{\alpha^*\mathbf{B}\alpha} = \mathbf{E}^*\mathbf{B}^{-1}\mathbf{E} > 0 \quad , \tag{15}$$

and the eigenvector α_{opt} is the optimum vector that can be calculated by

$$\alpha_{opt} = \mathbf{B}^{-1}\mathbf{E} \quad . \tag{16}$$

Equations (15) and (16) constitute the complete solution to the SRR-maximization problem.

Then, optimum RG can be calculated from

$$RG_{opt} = 10 \log_{10} (E^* \alpha_{opt}) \quad . \tag{17}$$

The details of this complete mathematical development, with proofs, can be found in existing material.[18]

EFFICIENT IMPLEMENTATION OF THE SRR MATHEMATICAL MODEL

PROCEDURE IN FLOW CHART

A large computer is necessary to carry out numerical implementation of such a complex problem. The complete computational procedure is shown as a flowchart in figure 2. Computational difficulties have been minimized in this procedure to reach a feasible solution.

A set of experimental computer programs has been developed in FORTRAN language at the Naval Underwater Systems Center (NUSC) in New New London on a Univac 1108 to implement the above procedure. The computer programs are not yet fully automated. Documentation of the detailed computerized model will be made available at a later date.

IMPORTANT CONSIDERATIONS

In order to construct an efficient computer model, certain important considerations cannot be neglected. These considerations may be applicable for developing all types of computer models. Several of these considerations are incorporated into the development of the SRR computer model; in fact, these become desirable features of this model. A brief discussion of these features follows.

Transportability

An effective computer model is expected to be used often and widely. It is desirable to develop computer programs not for one particular machine but for many different machines that speak the same dialect, but perfectly portable programs are not yet in existence. Nevertheless, program writers should attempt to develop programs in such a way that the least amount of effort is required to convert programs from one machine to the other. The SRR programs are designed in ANSI FORTRAN, which possesses this distinguishing quality, transportability.

Flexibility

Particular portions of the SRR programs are treated by special calculations. These calculations are performed either by means of existing

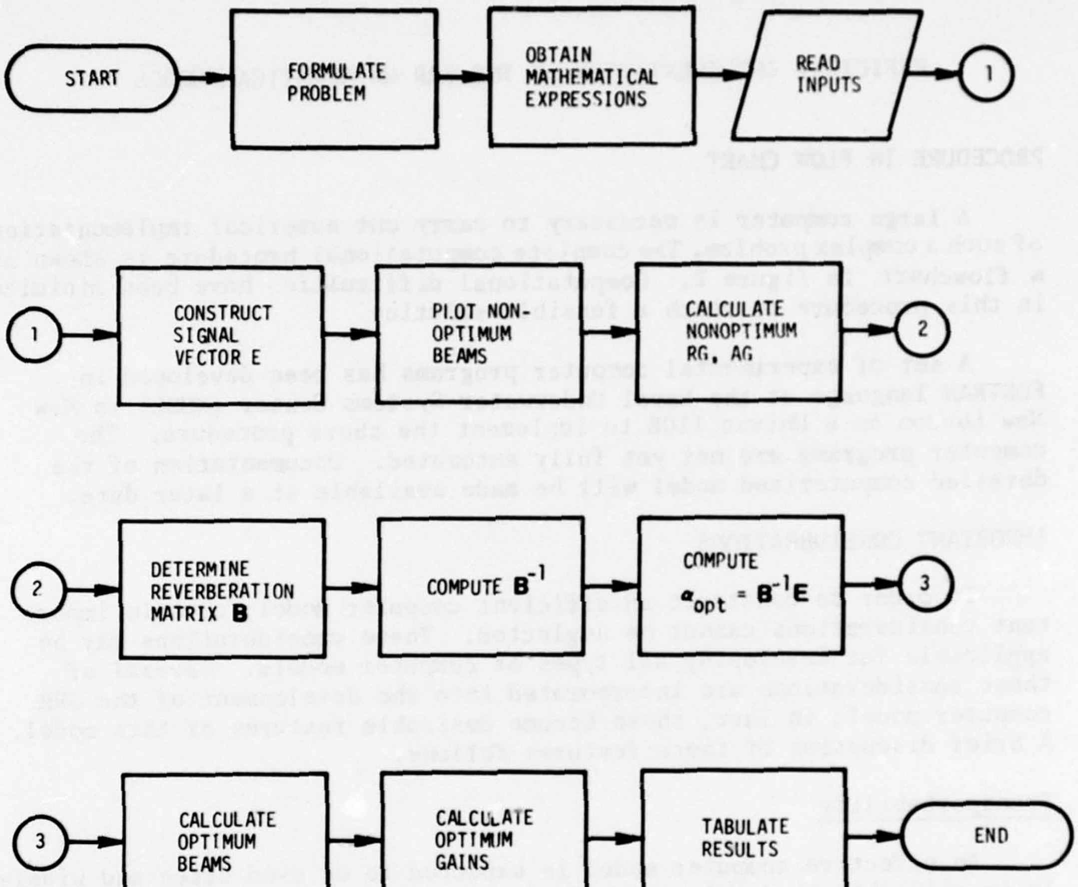

Figure 2. Computational Flow Chart

library packaged programs or by numerical techniques developed for this purpose. Generally speaking, these programs may not be the most efficient ones. The search for more efficient algorithms always goes on. To give certain flexibility, these SRR techniques are designed modularly in subroutine formats so that they can be distinguished and replaced more easily.

Accuracy and Speed

In carrying out numerical integrations, the step size plays an important role that determines the accuracy and the speed; one may be sacrificed to gain the other. However, accuracy is usually more important than speed. To meet less stringent accuracy requirements, speed can be improved by taking larger step sizes in the numerical procedures. High speed should not be accomplished by sacrificing accuracy beyond certain reasonable limits. One practical approach to improve the speed and still maintain good accuracy is to specialize the applications. The block Toeplitz[19] reverberation matrix is an example of a special application for equally spaced arrays.

Reliability

It frequently happens that a user fails to supply a piece of input; consequently, the program may fail or give poor results. To avoid this danger, all possible inputs are defaulted to reasonable values based on physical knowledge to maintain certain reliability in SRR programs.

PROBLEMS IN MODELING AND SOME PRACTICAL SOLUTIONS

PROBLEMS ENCOUNTERED

It is natural that in the process of modeling a solution for any real-world problem, other problems may arise. Although the problems encountered here are not unique, there are no simple and easy solutions to them. Some of the problems encountered in modeling a computerized mathematical solution to the SRR-maximization problem are outlined below in the order of consideration in a question-and-answer format, along with some practical solutions.

Q: The underwater acoustic problem of obtaining a mathematical expression for SRR involves three types of conditions; environmental, tactical, and equipment. Each condition involves a number of variables. It is almost impossible to come up with a mathematical expression that is general enough to cover all these variables. How should the general problem be approached to obtain a viable solution?

A: A practical approach is to simplify the problem in order to obtain a reasonable mathematical expression for SRR. If a simplified version cannot be solved, the solution to the general complicated problem can never be expected. Levels of higher complexity may be added after the less complex forms have been accepted as valid.

Q: How can this problem be simplified?

A: The problem may be simplified by making some reasonable, realistic assumptions so that the whole physical problem can be expressed mathematically. A starting point for an active-passive sonar system of n elements in a general environment would be to consider the passive case first, assuming omnidirectional array elements and a plane-wave signal in an isotropic noise field; then the directivity index (DI) can be expressed mathematically by a two-dimensional integral,[15] given by equation (6). From this, we begin by generalizing the definition of directive gain in order to establish the counterpart two-dimensional integral of DI for reverberation gain, RG (given by equation (3)), for active sonar systems. Then we demonstrate that the mathematical expression of equation (2) for RG was established with sufficient generality so that RI, AG, and DI are automatically accommodated.

Q: Why make these simple assumptions?

A: The fact is that if fewer simplifications are made, a more complicated mathematical expression may result. By the same token, if more simplifications are made, a simpler mathematical expression may result. Simple mathematical expressions are easier to handle. As long as the simplified mathematical expression is physically meaningful, this may be a good way to start the solution of a complex problem.

Q: Is this two-dimensional expression mathematically and physically valid?

A: The existence of the two-dimensional integral comes from the fact that the integrand is a bounded, continuous function in both θ and ϕ. The physical validity is confirmed by the actual experiments and measurements undertaken by sonar scientists.

Q: What are the problems in calculating the integral?

A: Problems are easier if the two-dimensional integral can be solved by closed-form solutions, for example, as in the case of an equally spaced line array in an isotropic noise field, where the solution is in the form $\frac{\sin x}{x}$. When closed-form solutions cannot be obtained, numerical integration can be used to evaluate the two-dimensional integral; a disadvantage is that the numerical integration may be very time-consuming if extreme accuracy is demanded or a large array is involved.

Q: Is the technique to maximize SRR in the mathematical model theoretically justified?

A: Since the physical property permits SRR to be expressible as a ratio of two quadratic forms, elegant quadratic-form theory to maximize SRR is applicable.

Q: What are the mathematical problems involved in obtaining the optimum RG?

A: Formulating a complete mathematical procedure to obtain optimum RG requires a great deal of matrix algebra and a thorough analysis of this generalized eigenvalue problem. Following the above, equation (8) has to be worked out for the determination of the signal matrix elements, and equation (11) has to be worked out for the determination of reverberation matrix elements. Finally, a solution to this generalized eigenvalue problem has to be worked out to obtain equation (17) for the calculation of optimum RG.

Q: Why is $\boldsymbol{\alpha}$ the optimum vector?

A: Maximizing SRR requires solving a generalized eigenvalue problem of the form $(\mathbf{A} - \lambda\mathbf{B})\boldsymbol{\alpha} = 0$. In the SRR model, because only one steering direction at a time is considered, the rank of $\mathbf{A} = 1$. Therefore, there exists only one nonzero, real λ whose associated eigenvector is $\boldsymbol{\alpha}$.

Q: What problems are encountered in obtaining $\boldsymbol{\alpha}_{opt}$ by $\mathbf{B}^{-1}\mathbf{E}$?

A: Since $\mathbf{B}$ is positive-definite, $\mathbf{B}$ is invertible. Solving $(\mathbf{A} - \lambda\mathbf{B})\boldsymbol{\alpha} = 0$ is the same as solving $\mathbf{B}(\mathbf{B}^{-1}\mathbf{A} - \lambda I)\boldsymbol{\alpha} = 0$. As long as $\mathbf{B}$ is not ill-conditioned, inversion programs are readily available. Because of small array element spacings, ill-conditioned $\mathbf{B}$ may result, in which case an accuracy problem is anticipated in inverting $\mathbf{B}$.

Q: Can RG always be improved?

A: The improvement of RG depends mainly on the element spacings. Some arrays, e.g., an equally spaced line array with ½λ spacing, are already in optimum state, in which case no improvement on RG can be expected.

Q: The SRR mathematical model presently in practice is developed to solve a restricted problem. How can the solution be generalized to solve the most general problem?

A: Since the reverberation power received by the array is always positive, the quadratic-form theory is still applicable. This mathematical model is still valid for the reason that a solution to the general

problem can be obtained through the use of measurements of the noise or reverberation matrix and the application of quadratic-form theory.

EXISTING PROBLEMS

In addition to the above problems, other problems exist that will be addressed, again, in a question-and-answer format.

Q: How can computations be done faster while maintaining reasonable accuracy?

A: A number of problems may be similar, such that they may have some complicated calculations in common. These complicated calculations need to be done only once and can be saved for repeated uses. For example, since the noise matrix $\mathbf{B}$ is not steering dependent, $\mathbf{B}$ and/or $\mathbf{B}^{-1}$ could be saved for an array being studied at different steering angles.

Q: Good results depend on accurate inputs. How can the accuracy of the inputs be ensured?

A: This model has no direct control of the inputs. Nevertheless, some inputs are defaulted meaningfully. The accuracy of the inputs is the responsibility of the user.

Q: How can a large array be handled when the reverberation matrix (or noise matrix) exceeds the computer capacity?

A: In a number of applications the Toeplitz technique can be used, requiring only that subsets of the entire matrix be stored because of repetition of the entries. When the reverberation matrix has Toeplitz structure but still exceeds memory, auxiliary storages are needed and the model has to be modified to handle additional I/O.

Q: What is a good method to handle the inversion of an ill-conditioned matrix?

A: There is no good way to handle the inversion of an ill-conditioned matrix. Since, when maximizing SRR, the problem is to find the eigenvector α of the problem $(\mathbf{A} - \lambda \mathbf{B})\alpha = 0$, the inversion of $\mathbf{B}$ can be avoided if an iterative technique is used.

The search for more effective answers to the questions presented still continues. Until effective solutions are found, the problems of accurate inputs, exceeding memory capacity, speed and accuracy, and solving a very large eigensystem will remain.

AN ILLUSTRATIVE EXAMPLE

The SRR model has been applied to several sonar systems. In this section an example using a cylindrical array of 192 elements has been selected for presentation. Computations were carried out in single-precision arithmetic on the NUSC, New London Laboratory, Univac 1108 computer with an allowable memory capacity of 65K. Compilation was carried out by means of the EXEC 8 operating system.

This array is selected for presentation for the following multiple purposes:

1. To show the validity of the SRR model, both in mathematics and in computer mechanism;

2. To exhibit and compare the nonoptimum and optimum beam patterns, as well as nonoptimum and optimum SNR;

3. To show how to overcome the difficulties and solve the problem efficiently, since this sizeable array encounters almost all the problems mentioned in the previous section;

4. To show that a general array of 192 elements may not be solvable due to the limit of memory capacity and existing support softwares or the impractical computation speed. However, this case has certain properties that enable it to be specialized to obtain a practical solution to the problem. The solution speed will be accelerated and the accuracy can still be maintained by this specialization; and

5. To show how the Toeplitz matrix can be used to advantage in this application.

ARRAY STRUCTURE AND INPUT CONDITIONS

To make calculations on this cylindrical array of 192 elements, some specific input conditions were assumed. The array consists of a total of 8 rows; each row has 24 elements. The radius of the cylinder is 8 feet. The horizontal angular separation is 5° between two adjacent elements and the vertical spacing between elements is 0.72 foot. Each element is assumed omnidirectional in an isotropic noise field. A plane wavefront is considered and the array is steered in the middle with a depression angle of 45°. A frequency of 3500 Hz is used.

PROBLEMS INVOLVED

In addition to minor problems, there are two major difficulties: the time required for computation and the limitation on computer memory.

In general, for an arbitrary array of 192 elements, generation of noise matrix elements will require $\frac{n(n-1)}{2} = 18336$ determinations. Each determination is the performance of a two-dimensional numerical integration in the region $0 \leq \theta \leq 2\pi$, $-\frac{1}{2}\pi \leq \phi \leq \frac{1}{2}\pi$. For acceptable accuracy, with a fixed mesh size (say 3° in each direction), 7200 integrand evaluations will be required. If each determination takes 1 second, total determination time will be about 5 hours and 6 minutes. The additional calculation of nonoptimum and optimum beam patterns, SNR, the matrix manipulation and inversion, and other factors, will push the total computation time up to about 6 hours to reach a solution. This lengthy computational time is a major difficulty.

The noise matrix will generally result in Hermitian form. Handling a 192 x 192 matrix of complex elements will exceed our present computer memory capacity, which is another major difficulty.

A PRACTICAL SOLUTION

An attempt was made to overcome the above difficulties by analyzing the properties of this array. A specialized approach to the problem may be used in order to obtain a practical solution by examining these properties.

In general, equation (11) is used to determine the elements b_{jk} of the noise matrix. A thorough analysis reveals that for omnidirectional elements in an isotropic noise field, the denominator of equation (11) is a two-dimensional integral that takes the form

$$\frac{1}{4\pi}\int_0^{2\pi}\int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} e^{i\frac{2\pi}{\lambda}[d_k - d_j]} \cos\phi \, d\phi \, d\theta \quad . \tag{18}$$

Since d_j is calculated by direction cosines, the above integral is evaluated in the form

$$\frac{1}{4\pi}\int_0^{2\pi}\int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} e^{i\frac{2\pi}{\lambda}[(x_k - x_j)\cos\phi\sin\theta + (y_k - y_j)\cos\phi\cos\theta - (z_k - z_j)\sin\phi]} \cos\phi \, d\phi \, d\theta . \tag{19}$$

A closed-form solution for the above integral has already been worked out and can be found in existing literature.[16] The solution is given by

$$(2\pi)^{3/2} J_{\frac{1}{2}}\left(\frac{2\pi}{\lambda} S\right) / \left(\frac{2\pi}{\lambda} S\right)^{\frac{1}{2}} \quad , \tag{20}$$

where

$$S = [(x_k - x_j)^2 + (y_k - y_j)^2 + (z_k - z_j)^2]^{\frac{1}{2}} \quad .$$

Since $J_{\frac{1}{2}}(kt) = \left(\frac{2}{\pi}\right)^{\frac{1}{2}} \sin(kt)/(kt)^{\frac{1}{2}}$, equation (19) can be simplified to the expression

$$\frac{\sin\left(\frac{2\pi}{\lambda} S\right)}{\left(\frac{2\pi}{\lambda} S\right)} \quad .$$

The exact solution also reveals that b_{jk} are real; therefore, the program can be specialized to handle a 192 x 192 symmetric matrix within our present memory capability. These simplifications also can reduce the 6 hour computation time to about 20 minutes.

NUMERICAL RESULTS

Following the procedure in the flowchart, we first calculate the nonoptimum beam patterns and nonoptimum DI. The nonoptimum horizontal beam pattern is given in figure 3 as a solid line. The nonoptimum DI is found to be 22.35 dB. We then proceed to calculate the optimum DI, which requires the determination of b_{jk}, the matrix inversion, and the calculation of optimum shading coefficients. The optimum horizontal beam pattern is also shown in figure 3, but as a dashed line. The optimum shading coefficients are given in tables 1 and 2. The optimum DI is found to be 22.87 dB, a gain of 0.52 dB.

The locations of the elements in Cartesian coordinates (X,Y,Z) are calculated using cylindrical coordinates; i.e., $X = r \cos\theta$, $Y = r \sin\theta$, $Z = Z$ for $32.5° \leq \theta \leq 147.5°$ at a θ-increment of 5°. Let W represent either X, Y, or Z; and let the indexes j, k, m be such that $1 \leq j,k \leq 24$, $m = 0,1,\ldots,7$. One can verify that $W_{j+24m} - W_{k+24m} = W_j - W_k$. This gives rise to a block Toeplitz noise matrix; each block is a 24 x 24 matrix. The total determination of the noise matrix elements is reduced to 4584 elements only, a saving of 13,752 locations. At the present time, a block Toeplitz inversion subroutine is not available. To handle the Toeplitz matrix, we determine the minimum necessary number of

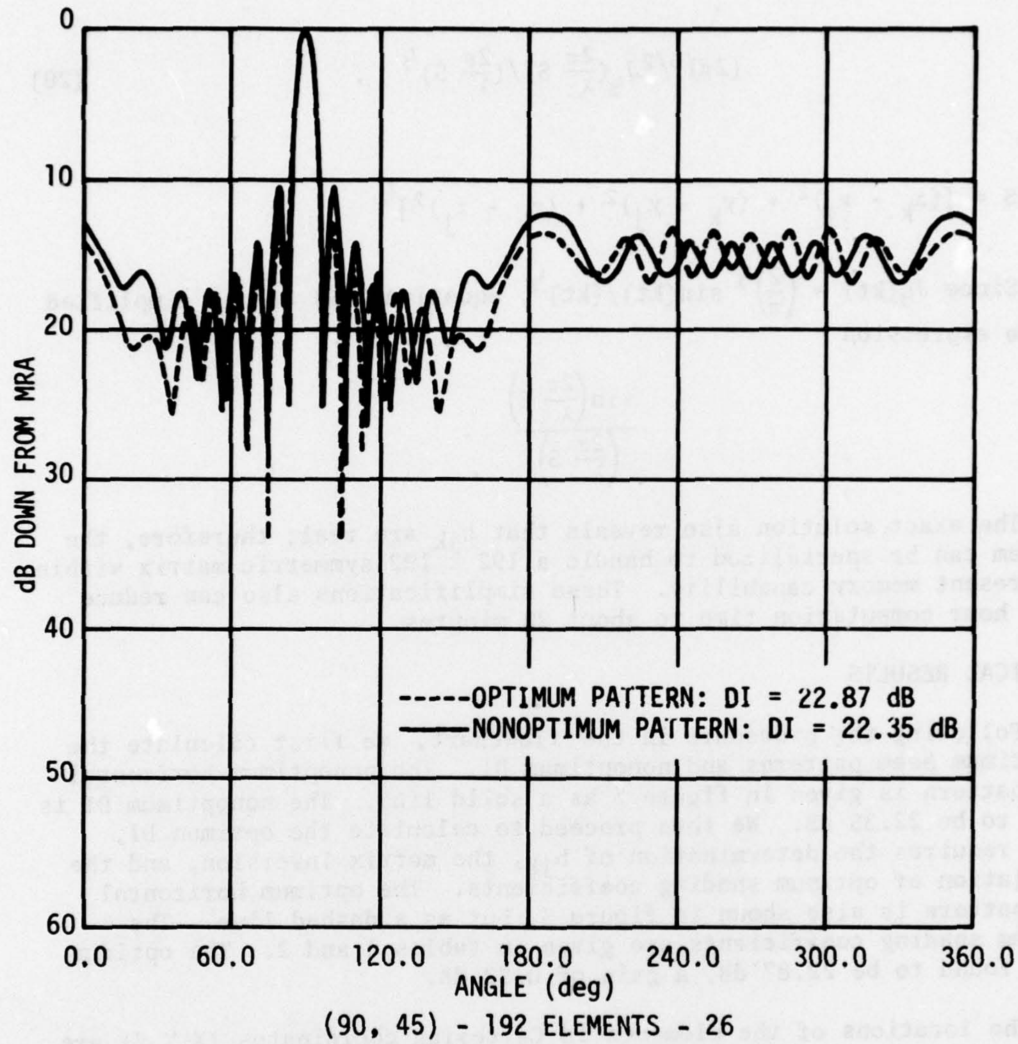

Figure 3. Horizontal Beam Pattern, Omnidirectional Elements

Table 1. Normalized Optimum Amplitude Coefficients*

.13280111+00	.82793002-01	.69882619-01	.20937963+00
.22840909+00	.79946042-01	.12954030+00	.18248319+00
.53000987-01	.13500443+00	.11804084+00	.10518964+00
.10518269+00	.11804765+00	.13499458+00	.53011604-01
.18247634+00	.12952937+00	.79948280-01	.22840452+00
.20937367+00	.69879678-01	.82793120-01	.13280116+00
.15796515+00	.20769910+00	.24633110+00	.38892712+00
.49766531+00	.38758016+00	.25325162+00	.36748828+00
.21688403+00	.16740339+00	.18954971+00	.73089208-01
.73088109-01	.18953667+00	.16736566+00	.21686434+00
.36745383+00	.25321673+00	.38755613+00	.49764040+00
.38890715+00	.24632018+00	.20769469+00	.15796425+00
.20082618+00	.38681863+00	.51691626+00	.64043237+00
.79061432+00	.75863105+00	.52921947+00	.57166698+00
.48378490+00	.23120761+00	.30113671+00	.44181563-01
.44248821-01	.30107630+00	.23113395+00	.48372232+00
.57159514+00	.52915861+00	.75857050+00	.79056159+00
.64039297+00	.51689137+00	.38680651+00	.20082197+00
.23281741+00	.49893874+00	.70143386+00	.80714849+00
.94323532+00	.10000000+01	.76629516+00	.66983428+00
.66032016+00	.29791722+00	.34651732+00	.82609018-01
.82646132-01	.34641811+00	.29782193+00	.66022555+00
.66972719+00	.76620273+00	.99991325+00	.94316290+00
.80709314+00	.70139802+00	.49892030+00	.23281107+00
.19501743+00	.47937338+00	.72973312+00	.81636749+00
.86397782+00	.95812562+00	.80458716+00	.59177101+00
.64975729+00	.33425427+00	.29288882+00	.13482484+00
.13480165+00	.29277998+00	.33415643+00	.64964522+00
.59166194+00	.80448604+00	.95803249+00	.86390107+00
.81631014+00	.72969721+00	.47935589+00	.19501313+00
.11788686+00	.34527393+00	.58367705+00	.65314620+00
.59562206+00	.66302999+00	.63679373+00	.39407163+00
.45895585+00	.30569579+00	.16204624+00	.15030185+00
.15024535+00	.16197544+00	.30562235+00	.45887188+00
.39399029+00	.63671771+00	.66296146+00	.59556410+00
.65310303+00	.58365177+00	.34526428+00	.11788674+00
.62190854-01	.18294750+00	.34538620+00	.40973934+00
.30626509+00	.28358918+00	.37137629+00	.20994914+00
.19662871+00	.21663738+00	.34719955-01	.12699706+00
.12695771+00	.34734902-01	.21659505+00	.19659228+00
.20990735+00	.37133591+00	.28355505+00	.30623423+00
.40971760+00	.34537617+00	.18294634+00	.62191825-01
.72522793-01	.79662835-01	.17091981+00	.23342725+00
.16994681+00	.19371152-01	.15298356+00	.14635226+00
.56496797-01	.13473249+00	.97175206-01	.11101475+00
.11100784+00	.97182458-01	.13472130+00	.56508338-01
.14634013+00	.15297552+00	.19381611-01	.16993779+00
.23342173+00	.17091950+00	.79663160-01	.72522303-01

*In row order.

Table 2. Normalized Optimum Phase Coefficients*

.12854919+01	.49387090+01	-.47052112+00	.32125690+01
.80836147+00	.47209648+01	-.46518061+00	.38068202+01
.28251857+01	.29809950+01	.20421724+01	.21630821+01
.21630404+01	.20422223+01	.29810061+01	.28251920+01
.38068703+01	-.46519712+00	.47210894+01	.80839306+00
.32125762+01	-.47053364+00	.49387130+01	.12854825+01
.35489694+01	.54682299+00	.33924749+01	.63557684-01
.34180756+01	.54910171+00	.32653424+01	.16945398+00
.38884690+01	-.22880358+00	.39626631+01	.44318333+01
.44313324+01	.39628450+01	-.22878817+00	.38886137+01
.16950333+00	.32653958+01	.54916185+00	.34181024+01
.63572198-01	.33924984+01	.54684073+00	.35489718+01
-.17386448+00	.31979584+01	.56545347-01	.31003526+01
.15524596-01	.33750664+01	.20219657+00	.31234733+01
.34980986+00	.30915414+01	.24379456+00	-.57012856-01
-.57514042-01	.24393511+00	.30916221+01	.34990495+00
.31235075+01	.20225623+00	.33751057+01	.15543014-01
.31003644+01	.56558222-01	.31979665+01	-.17386428+00
.24623131+01	-.34038785+00	.29765801+01	-.13785109+00
.29917011+01	.00000000	.32453916+01	-.14612001+00
.32253896+01	.10509497+00	.30279628+01	.16217330+01
.16229149+01	.30280263+01	.10523209+00	.32254537+01
-.14610034+00	.32454392+01	.24110079-04	.29917095+01
-.13784358+00	.29765835+01	-.34039235+00	.24623001+01
.50968453+01	.24662596+01	-.38773412+00	.28891855+01
-.28782260+00	.29277679+01	-.25810927-01	.29196191+01
-.14921746+00	.33210039+01	-.42176142+00	.41287593+01
.41294898+01	-.42180148+00	.33211495+01	-.14919665+00
.29196350+01	-.25782436-01	.29277723+01	-.28782484+00
.28891854+01	-.38774121+00	.24662427+01	.50968158+01
.13089936+01	-.10508331+01	.24907595+01	-.38131097+00
.27692373+01	-.42415571+00	.29654022+01	-.20916614+00
.27330940+01	.15716442+00	.23393358+01	.47723716+00
.47759348+00	.23390914+01	.15728939+00	.27330796+01
-.20915961+00	.29654120+01	-.42417505+00	.27692279+01
-.38131922+00	.24907413+01	-.10508618+01	.13089531+01
.33542584+01	.14905351+01	-.10180405+01	.26354111+01
-.26036361+00	.24877763+01	-.42787129+00	.31132537+01
-.84244648+00	.31406809+01	.40480070+01	.29940732+01
.29941207+01	.40468896+01	.31407327+01	-.84256625+00
.31133039+01	-.42788979+00	.24877248+01	-.26036507+00
.26353973+01	-.10180700+01	.14905012+01	.33542486+01
.48997399+01	.31972951+01	.13941491+01	-.85492179+00
.32418062+01	.24888511+01	.20045682+01	.22169605+00
.14879328+00	-.55421704+00	.51622863+01	.51416783+01
.51416118+01	.51623651+01	-.55423763+00	.14870131+00
.22173604+00	.20045041+01	.24890826+01	.32418243+01
-.85493872+00	.13941286+01	.31972921+01	.48997233+01

*In row order.

elements and set up the complete Toeplitz matrix by a simple program loop, then perform the inversion by Gauss-Jordan reduction subroutine.

In reality, the individual directionality of the elements should be taken into consideration. Incorporating $R_j(\theta,\phi)$ into the integrand not only makes it difficult to obtain a closed-form solution but also increases the computation time tremendously. To obtain each b_{jk} through numerical integration with element directionality will take at least three times longer than the omnidirectional case, thus requiring about 18 hours computer time to reach a solution. This time requirement almost makes the solution impossible. At present, we use the optimum coefficients, obtained from the case of omnidirectional elements, to process for an improvement. Numerical results show an improvement of 0.28 dB. However, this answer remains to be justified theoretically. A set of nonoptimum and improved beam patterns of directional elements is shown in figure 4 with the associated DI values.

CONCLUSIONS

Even though certain simplifications have been made on the actual physical conditions in order to develop the SRR model, the model works for both passive and active sonar systems in general. This model determines a set of coefficients that not only can be used for the maximization of SRR, which leads to the improvement of sonar performance of existing sonars, but also can be used to guide the design of future sonars.

When modeling a solution for any complicated problem, it is desirable to begin with reasonable simplifications of the actual problem to make it solvable by a simple mathematical model with potentiality for generalization. When a mathematical expression does not have closed-form solutions, it should be solvable by some numerical technique. It is most desirable that the solution be either unique or optimum. If SRR can be maximized, the SRR model will produce optimum results.

It may seem impossible to construct a general model to handle any situation pertinent to a complicated problem. Then it is possible to specialize the model for particular applications so that advantage can be taken of special properties. The SRR model has been specialized for a number of applications. Numerical results and speed show great efficiency in this model.

In designing a computer model, it is desirable to automate the interactions internally so that the users are not bothered. Some price must be paid to achieve this automation. Whether or not it is worth the overhead to achieve the general product has to be weighed carefully

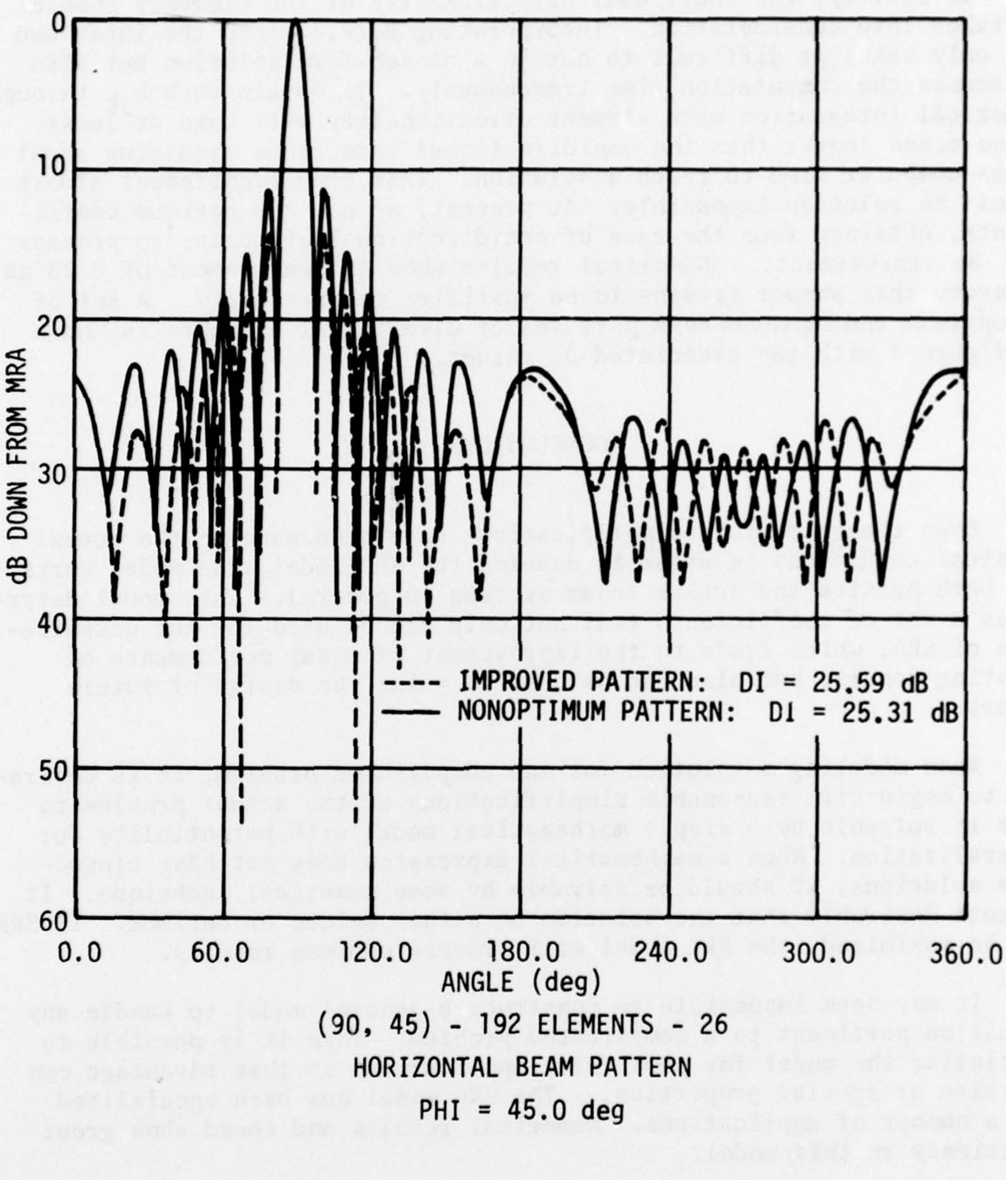

Figure 4. Horizontal Beam Pattern, Directional Elements

by the designer. The SRR model, as illustrated in the flowchart (figure 2), is performed in three stages with little user intervention between stages.

Modeling a computerized mathematical solution for any problem is an art. Besides accuracy, speed, and mathematical validity, a model must also try to be short and simple, as general as possible, flexible and reliable, and easy to use in order to be considered efficient.

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